Written Exam for the B.Sc. or M.Sc. in Economics winter 2015-16

Micro III

Final Exam

Date: 21 December 2015

(2-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam question consists of 4 pages in total (including the front page)

PLEASE ANSWER ALL QUESTIONS.
PLEASE EXPLAIN YOUR ANSWERS.

1. Consider the following game G.

		Player 2		
		L	C	R
	U	6, 3	3, 0	0, 1
Player 1	M	5, 4	1, 7	4, 3
	D	2, 5	5, 7	0, 0

   (a) Solve the game by iterated elimination of strictly dominated strategies. If you get a unique solution, indicate this. If your solution is not unique, write up the reduced game where you have eliminated the strictly dominated strategies.

   (b) Find all the pure and mixed-strategy Nash Equilibria.[1]

   (c) It has been argued that randomization in decision making lacks 'behavioral support'. Give one (and just one) example of a different interpretation of mixed strategies, that does not rely on players actually randomizing.

   (d) Suppose we repeat the game twice. Let the new game be denoted $G(2)$. Find a Subgame-perfect Nash Equilibrium of $G(2)$ and write it up formally. (Any equilibrium will do, you do NOT have to find all equilibria.)

2. Two tech entrepreneurs have made 1 dollar from selling a new app and need to decide how to allocate the gains. If no agreement is reached, neither entrepreneur gets anything. Let x_1 and x_2 be the amounts that entrepreneur 1 and 2 get in the negotiation. Their utilities are:

$$u_1(x_1) = 4x_1$$
$$u_2(x_2) = 2\sqrt{x_2}.$$

   (a) Can the axioms Pareto efficiency (PAR), Symmetry (SYM) and Invariance to equivalent payoff representations (INV) be used to conclude that the Nash Bargaining Solution must satisfy $v_1^* = v_2^*$? Explain briefly.

   (b) Find the Nash Bargaining Solution. What are the allocations?

   (c) Now, suppose the entrepreneurs have signed a contract before they started the venture, guaranteeing that in case of disagreement, entrepreneur 1 gets to keep 0.5 dollar whereas entrepreneur 2 gets nothing. Find the Nash Bargaining Solution. What are the allocations?

   (d) Compare the answers in (b) and (c). If the allocations are the same, explain why this is the case. If they are different, explain why this is the case.

[1] For the mixed-strategy equilibria, you can assume that player 1 plays U with probability p_1, M with probability p_2 and D with probability $1 - p_1 - p_2$. Similarly, assume that player 2 plays L with probability q_1, C with probability q_2 and R with probability $1 - q_1 - q_2$.

3. Consider the entry game represented in Figure 1, in which the incumbent can be *weak* ($i = w$) or *strong* ($i = s$). Here, the incumbent **does not** know his own type, but the outsider does.[2] You can think of this as a probability θ that the outsider has found the incumbent's 'weak spot'. Suppose $\theta \in (0, 1)$.

   The timing of the game is as follows. The outsider must first decide whether to enter (E_i) or not (N_i). (Here the i indicates the type of the incumbent, since the outsider conditions his choice on the incumbent's type.) If he doesn't enter, we assume that the game ends. If he enters, on the other hand, the incumbent can choose either to fight (F) or acquiesce (A). If he acquiesces, the game ends. If he fights, the game continues. In this case, the outsider must decide whether to stay (S_i) or leave (L_i). (Again, $i = w, s$.)

   Suppose the incumbent's beliefs in his information set attach probability p to him being the weak type. The payoffs are as indicated in Figure 1. *The first payoff is that of the incumbent, the second is that of the outsider.*

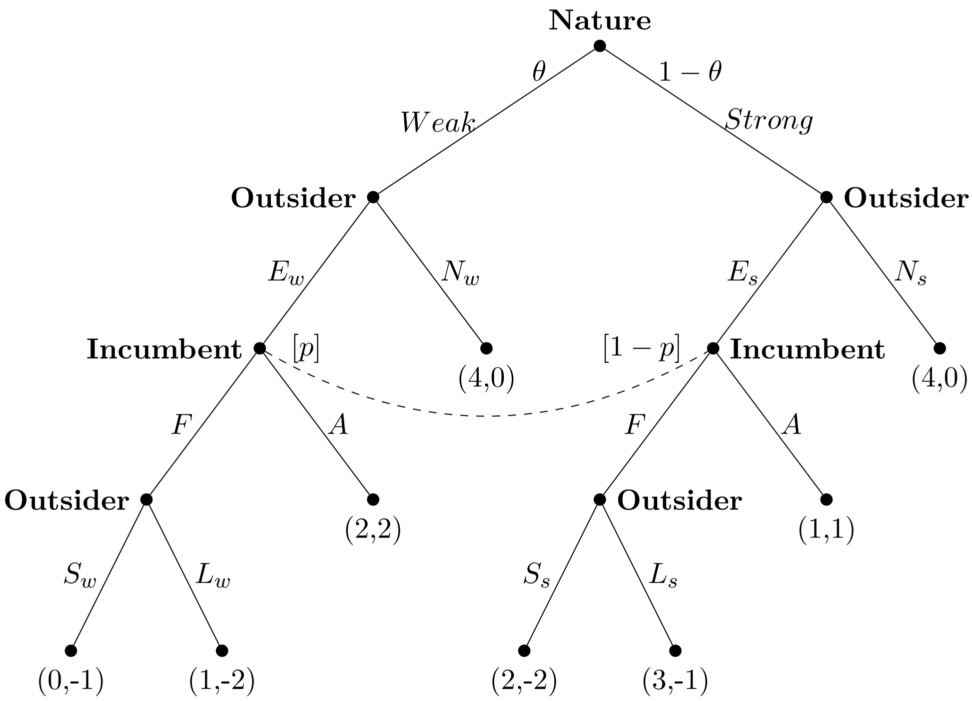
   (a) Indicate how many strategies each player has, and write up one such strategy for each player. Is this a game of imperfect or incomplete information?

   (b) Show that for certain values of θ, there is an equilibrium in which the outsider always enters (plays E_i for $i = w, s$) and the incumbent acquiesces (plays A). Be careful to specify how the equilibrium depends on p and θ. (Hint: Use Bayes' Rule to calculate p given θ and given that the outsider always enters.)

   (c) Show that there is also an equilibrium in which the outsider never enters (he plays N_i for $i = w, s$). Be careful when you write up the equilibrium to specify p, and how the equilibrium depends on θ. (Hint: In this case, Bayes' Rule does not apply to p.)

   (d) Consider the equilibrium in (c) where the outsider never enters. Does it satisfy SR5 ('strict domination')?

Figure 1: Entry game



[2]Notice that this is the 'opposite' of the entry game you saw in the lectures.

4. Consider a *first-price sealed bid auction* with two bidders, who have valuations v_1 and v_2, respectively. For $i = 1, 2$, these values are distributed independently uniformly with

$$v_i \sim u(1, 2).$$

Thus, the values are *private.*

(a) Suppose player j uses the strategy $b_j(v_j) = cv_j + d$. For $i \neq j$, show that conditional on this strategy, the probability that i wins when he bids b_i is

$$\mathbb{P}(i \text{ wins}|b_i) = \frac{b_i - d - c}{c},$$

whenever $c + d \leq b_i \leq 2c + d$.

(b) Using the result in (a), show that there is a symmetric Bayesian Nash Equilibrium in linear strategies: $b_i(v_i) = cv_i + d$, $i = 1, 2$. Find c and d.